Bihar Mathematical Society

Talent Nature Programme (TNP) 2021 (Level II & III)

Full Marks:- 100 **Time: $2\frac{1}{2}$ Hours**

Answer all questions. All questions carry equal marks.

1. Let R be a principal ideal domain, and F be a free R-module with a basis consists of n elements. Then any sub-module K of F is also free with a basis consisting of m elements, such that $m \le n$.
2. Every Simple module is non-zero.
3. Let $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be the series of non-negative terms. Let $k > 0$ be a constant and m be a fixed positive integer, then if $a_n \le k b_n \ \forall \ n \ge m$ and $\sum_{n=1}^{\infty} b_n$ is convergent, then $\sum_{n=1}^{\infty} a_n$ is convergent.
4. Let M be free R-module with basis $\{e_1, e_2, \ldots, e_n\}$. Then $M \simeq R^n$.
5. Expand $4x^2 + 5x + 3$ in powers of $(x-1)$ by using Taylor series.
6. Find the necessary and sufficient conditions for the function $w = f(z) = u(x, y) + iv(x, y)$ to be analytic in given region R, are
   (i) $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ are continuous functions of x and y in the region R.
   (ii) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ or $u_x = v_y, u_y = v_x$.(Cauchy-Riemann equations or C-R equations)
7. Let $\phi : F^3 \to F^2$ be linear mapping given by $\phi(a, b, c) = (a + b + c, b + c)$. Find the matrix A of ϕ w.r.t. the standard bases of F^3 and F^2. Also, find the matrix A' of ϕ w.r.t. the bases
   $\mathcal{B}' = \{(-1,0,2), (0,1,1), (3,-1,0)\}$ and $\mathcal{C}' = \{(-1,1), (1,0)\}$ of F^3 and F^2, respectively. Verify that $A' = P^{-1}AQ$, where P and Q are the matrices of transformations.
8. Verify Gauss divergence theorem for $\vec{F} = (x^2 - yz)\hat{i} - (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ taken over the rectangular parallelopiped $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$.
9. Let f be an R-homomorphism of an R-sub-module M onto an R-module N. Then prove that $\frac{M}{Ker\ f} \cong N$.
10. Let A is an R-module of M and B is an R-module of N. Then prove that $\frac{M \times N}{A \times B} \cong \frac{M}{A} \times \frac{N}{B}$